

Algorithm for determination of the damage characteristic (C-S) curve of asphalt mixtures

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ABSTRACT

This paper presents the algorithm for calculating the damage characteristic curve obtained in direct tension tests taking into account sinusoidal controlled strain loading. The Viscoelastic Continuum Damage formulation is presented in a summarized form for the algorithm, in which the pseudo strain, at the instants associated to the observed stress, is calculated using the expression of the linear viscoelasticity stress under controlled strain testing. This facilitates subsequent treatment of the data to obtain the C vs. S curve. The proposed algorithm is simple to understand and easy to implement computationally. The algorithm was validated with the results of fatigue test simulations in three mixtures, which have indicated its potential.

Keywords:

Asphalt Mixture,

Fatigue,

Viscoelastic Continuum Damage

Model,

C vs. S

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1. INTRODUCTION

Fatigue cracking is one of the major distresses in asphalt pavements. The cause of these cracks, which are affected by repeated (i.e., cyclic) loading over time, can be tied to aspects such as weak pavement sublayers, insufficiently designed asphalt materials, or changes in strain tolerance of the mixture due to field aging.

There are different approaches to quantify the fatigue crack process in asphalt mixtures. Among the different approaches, Nascimento (2015) proposed the use of the Viscoelastic Continuum Damage (VECD) for characterizing Brazilian asphalt mixes in terms of fatigue cracking. The use of the continuum damage approach to study fatigue cracking in asphalt mixtures was initiated by Kim and Little (1990), who presented the behavior of cyclic tests on a prismatic

sample as well the benefits of rest periods. Continuum damage mechanics ignores specific microscale details and attempts to characterize the material by considering the effect of microstructural changes on observable macro properties (Lemaitre, 1996). It considers the effect of damage in the constitutive modeling of viscoelastic materials by quantifying two variables: the damage parameter (S) and the pseudo stiffness (C). The damage parameter, in general, quantifies any change in the microstructure that results in a reduction of stiffness.

Fatigue curve is a characteristic of asphaltic materials that can be used to predict the durability of asphalt surface courses through computer simulation. It is obtained through laboratory fatigue tests, which measure the integrity of the specimen, represented by the stiffness value over time. Kim (2009) developed an algorithm for adjusting the damage characteristic curve (i.e., the C vs. S curve) of asphalt mixtures from the stresses observed in controlled strain tests performed on laboratory specimens.

The algorithm presented herein is an improvement of the referred algorithm. The original algorithm proposes to arbitrate the increment of damage, while the new algorithm proposes to arbitrate the initial damage (S_0). The original algorithm calculates the rate of change of integrity with damage growth, incrementally, whereas the proposed algorithm calculates through the derivative of the equation that relates integrity with damage parameter. The proposed algorithm calculates the pseudo-strains from the LVE theoretical stresses due to the applied strains.

This paper is organized as follows: Section 2 presents the theoretical basis of formulation; Section 3 presents the proposed algorithm; Section 4 shows results of three mixtures analyzed with the proposed algorithm; and Section 5 presents the conclusions.

2. THEORETICAL BASIS

The damage characteristic curve represents the fundamental relationship between damage and pseudo stiffness (material integrity) for asphalt mixtures. Combined with the linear viscoelastic properties of asphalt concrete, the damage characteristic curve can be used to analyze the fatigue characteristics. This section presents a brief theoretical basis for obtaining the damage characteristic curve.

2.1. Linear Viscoelasticity

Viscoelastic materials are those for which the relationship between stress (σ) and strain (ε) depends on time (t). Linear viscoelasticity is a theory describing the behavior of such ideal materials. Linear viscoelastic (LVE) materials exhibit time and temperature dependent behavior, and their current response is dependent on both the current input and all past input (i.e. the input history). By contrast, the response of an elastic material is only dependent on the current input. Constitutive relationship for LVE materials is typically expressed in the convolution integral form, as follows:

$$\sigma(t) = \int_0^t E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau \quad (1)$$

where $E(t)$ is the relaxation modulus. The analytical forms for this function is generally given by Prony series:

$$E(t) = E_\infty + \sum_{i=1}^m E_i e^{-\frac{t}{\rho_i}} \quad (2)$$

where: E_∞ , E_i are stiffness constants and ρ_i are relation times.

The formulation through Prony series has the advantage of being linked to the behavior of the Maxwell-Wiechert mechanical model as shown in Figure 1. Note that the characterization of LVE behavior is undertaken by performing temperature and frequency sweep tests which are then processed to obtain the series coefficients.

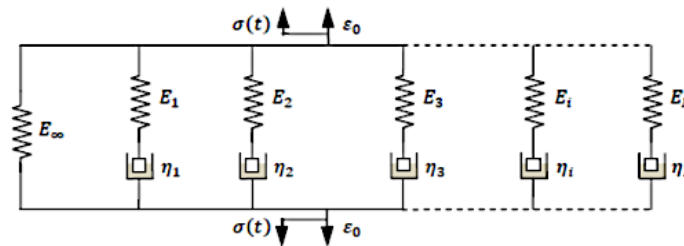


Figure 1. Wiechert (or Generalized Maxwell) model

2.2. Elastic-Viscoelastic Correspondence Principle in Terms of Pseudo-Variables

The theory of linear viscoelasticity and some nonlinear viscoelasticity models have been established in the past (Pagen, 1965; Ferry, 1980; Lamborn and Schapery, 1993; Park et al., 1996). However, solution of various kinds of viscoelastic boundary value problems still remains a complex problem. Fortunately, the nature of the correspondence existing between the elastic and the integral-transformed (e.g. Laplace-transformed or Fourier-transformed) viscoelastic field equations for linear non-aging materials enables one to obtain a solution to a viscoelastic boundary value problem from the solution of its corresponding elastic boundary value problem (Park et al., 1996).

Originally proposed by Schapery (1984), the elastic-viscoelastic correspondence principle in terms of pseudo-variables involves the use of parameters which do not necessarily represent physical quantities such as strains and stresses, but which are useful in transforming the viscoelastic stress-strain relationships into elastic-like equations.

On the other hand, it has been shown that the stress-strain relations for a broader class of viscoelastic materials can be represented by elastic-like equations through the use of so-called pseudo-variables. This simplifying feature enables correspondence principles to be established and applied to linear and nonlinear viscoelastic boundary value problems, with or without aging (Schapery, 1981; Schapery, 1984). The Laplace or Fourier transform are not used in this approach.

Using this correspondence principle one can obtain viscoelastic solutions from their elastic counterparts through a simple substitution and integration. This principle is applicable to stationary or time-dependent boundary conditions and does not require a transform inversion step, but it rather requires only the evaluation of a convolution integral (Kim, 2009; Nascimento, 2015).

The constitutive law for a viscoelastic material can be expressed in a form similar to Hooke's law of linear elasticity for all times. Using pseudo strain in place of physical strain, the uniaxial constitutive relationship presented in Equation (1) can be rewritten as follows:

$$\sigma(t) = E_R \varepsilon^R \quad (3)$$

where E_R is a particular reference modulus, typically taken as one, and ε^R is the pseudo strain:

$$\varepsilon^R = \frac{1}{E_R} \int_0^t E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau \quad (4)$$

Therefore, pseudo strain can be defined as the numerical value of the stress (without damage) divided by E_R .

2.3. Work Potential Theory

The Work Potential Theory (WPT) damage model is based on linear viscoelastic characterization and nonlinear damage accumulation. Schapery's WPT Theory (Schapery, 1994) contains parameters that characterize the variation of material integrity with damage growth. To clarify the physical significance of these parameters, a step-by-step process is followed to link the damage functions to the parameters.

The pseudo strain energy density function of the material is given by:

$$W^R = \frac{1}{2} C(S) (\varepsilon^R)^2 \quad (5)$$

where the pseudo stiffness C is a function of a damage parameter S .

Then, stress is obtained from derivation of the pseudo strain energy:

$$\sigma = \frac{\partial W^R}{\partial \varepsilon^R} = C(S) \varepsilon^R \quad (6)$$

According to Schapery (1994), the following form can be used to describe damage evolution in a viscoelastic material:

$$\dot{S} = \left(-\frac{\partial W^R}{\partial S} \right)^\alpha \quad (7)$$

where \dot{S} is the damage evolution rate over time, W^R is the pseudo strain energy density function, and α is a material constant. According to Park *et al.* (1996), the constant α is based on LVE fracture mechanics. Its value is related to the viscoelastic characteristics of the material established by its creep flow or its relaxation modulus. Given the α value, the parameters of the material damage curve can be obtained from the stresses observed in the fatigue test. Hence, any deviation in output (stress or strain) from the theoretical value is reflected by Equation (6). Consequently, stress-pseudo-strain relationships provide a correspondence between linear viscoelastic undamaged and damaged bodies. All damage growth over time is depicted in a reduction in material stress.

The evolution law in Equation (7) reflects rate-effects through ε^R in W^R and explicit time-effects through the time-derivative of S on the left hand side. The only damage-related constitutive parameters to be determined for a given material are the function $C(S)$ and the constant α . The validity of this assumption will be evaluated by a comparison of the algorithm with the experimental observation of response of the specimens.

2.4. Viscoelastic Continuum Damage Theory

The continuum damage mechanics, originally developed for elastic materials, can be generalized for viscoelastic materials using the elastic-viscoelastic correspondence principle (Kim, 2009). The VECD model combines elements from the preceding sections to arrive at the constitutive relationship. From continuum damage, the stiffness reduction is defined by the pseudo stiffness.

The WPT uses an internal state variable (S) to quantify damage. This internal state variable quantifies any microstructural changes that result in the observed stiffness reduction.

For asphalt concrete under tensile stress, this damage variable is related primarily to the microcracking phenomenon.

The relationship between C and S is a material property that is independent of the mode of loading, temperature, and load amplitude and is referred as the damage characteristic curve (Kim, 2009). With these considerations, the nonlinear constitutive relationships are given by:

$$\sigma = C(S)\varepsilon^R \quad (8)$$

The pseudo stiffness (C) decreases from the value of 1 as the accumulated damage increases. According to Kim (2009), the expression for $C(S)$ can be represented by a power law as:

$$C(S) = 1 - C_{11} S^{C_{12}} \quad (9)$$

where C_{11} and C_{12} are regression constants that fit the curves obtained in the fatigue tests of the material. These constants characterize the material with respect to its susceptibility to damage.

Substituting $\frac{dC}{dS}$ into the Work Potential Equation by the derivative of Equation (9), and $\frac{dS}{dt}$ by its incremental form, $\frac{\Delta S}{\Delta t}$, one obtains:

$$S_{n+1} = S_n + [1/2 C_{11} C_{12} S_n^{(C_{12}-1)} (\varepsilon_n^R)^2]^\alpha \Delta t \quad (10)$$

where, n and $n + 1$ are the steps at times t_n and t_{n+1} , respectively; S_n and S_{n+1} are damage parameter at n and $n + 1$, respectively; Δt is the time interval between t_n and t_{n+1} .

Equation (10) can be used to determine the grown of damage parameter in asphalt mixture. This parameter along with material pseudo stiffness will be used to obtain C vs. S curve, where:

$$C_{n+1} = \frac{\sigma_{n+1}}{\varepsilon_{n+1}^R} \quad (11)$$

where σ_{n+1} is the stress observed at time t_{n+1} and ε_{n+1}^R is the pseudo strain at time t_{n+1} .

3. METHODOLOGY

Viscoelastic damage characterization refers to the determination of the characteristic damage relationship (i.e. the C vs. S curve). This paper presents a method for calculating the C vs. S curve obtained in direct tension tests considering sinusoidal controlled strain loading.

3.1. Sinusoidal Controlled Strain Rate Test

In this test, a controlled sinusoidal strain is applied while the stress response is measured. The strain loading is given by:

$$\varepsilon(t) = \frac{\varepsilon_0}{2} \text{sen}(\omega t) \quad (12)$$

where ω is the angular frequency of the action and ε_0 is the corresponding strain amplitude (peak to peak). The stress response of a linear viscoelastic material to this sinusoidal loading is given by (Almeida, 2019):

$$\sigma(t) = \frac{\varepsilon_0}{2} \left[E_\infty \text{sen}(\omega t) + \omega^2 \sum_{j=1}^n \frac{E_j \rho_j}{(1 + \omega^2 \rho_j^2)} \left(\text{sen}(\omega t) + \frac{\cos(\omega t)}{\omega \rho_j} - \frac{e^{-\frac{t}{\rho_j}}}{\omega \rho_j} \right) \right] \quad (13)$$

The test is shown schematically in Figure 2.

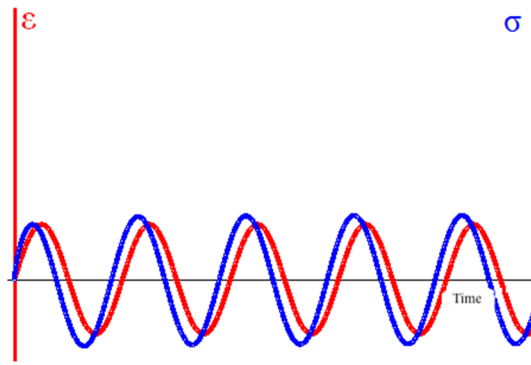


Figure 2. Sinusoidal Controlled Strain Rate Test

3.2. Algorithm to Obtain Parameters C_{11} and C_{12} of the C vs S curve

The proposed algorithm is based on Equations (4), (9), (10), (11) and (13). It is assumed that the viscoelastic characterization of the asphalt mixture has been held previously, i.e., the values of the Prony series constants and the α value of the asphalt mixture are known.

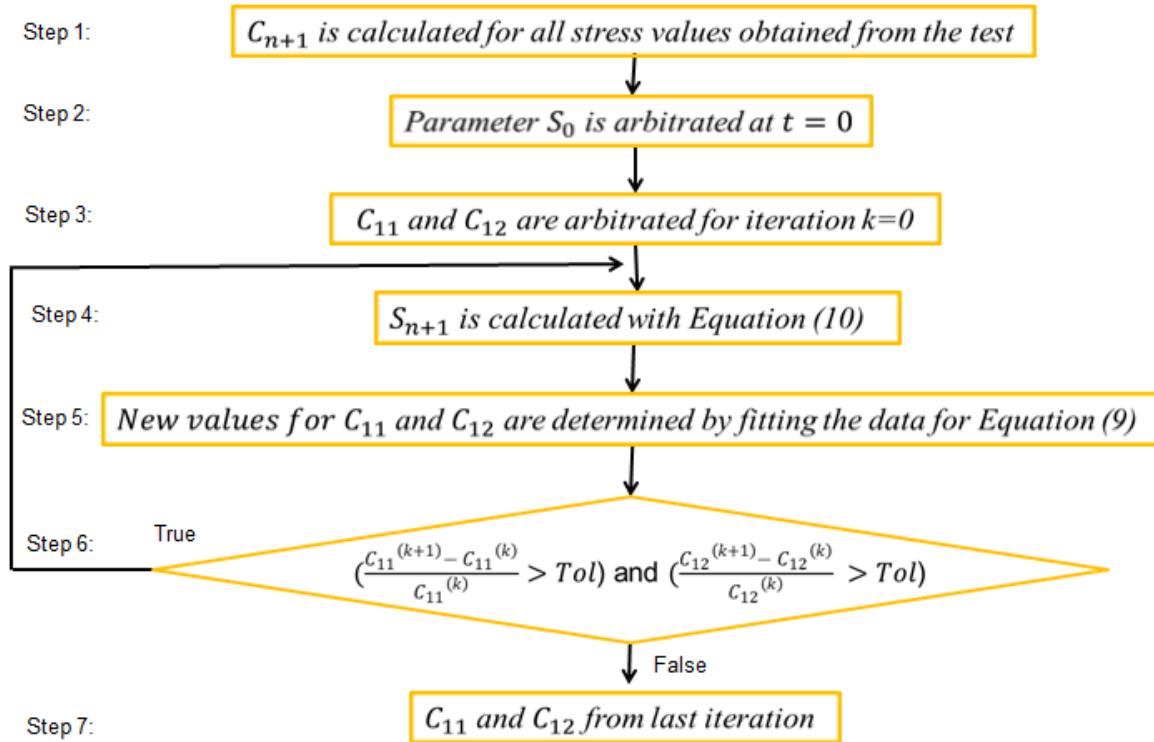


Figure 3. Algorithm flowchart

The algorithm consists of seven major steps presented in Figure 3. Step 1 corresponds to the computation of the pseudo stiffness (C_{n+1}) for each measured stress (σ_{n+1}), with the pseudo strain (ϵ_{n+1}^R) evaluated dividing corresponding viscoelastic stress, computed using Equation (13), by E_R according Equation (4). The initial damage parameter (S_0) and the initial parameters $C_{11}^{(0)}$ and $C_{12}^{(0)}$ chosen by the user, are prescribed in Steps 2 and 3, respectively. The damage parameter for each time step (S_{n+1}) is evaluated in Step 4 using the current values of parameters $C_{11}^{(k)}$ and $C_{12}^{(k)}$. In Step 4, a set of ordered pairs ($S^{(k)}, C^{(k)}$) for iteration k are fitted

to Equation (9) using the Least Squares Method, obtaining new values for the following iteration: $C_{11}^{(k+1)}$ and $C_{12}^{(k+1)}$. Then, the algorithm returns to Step 4 with the new values of C_{11} and C_{12} , repeating the procedure. The iterations stop when the relative difference between the values of C_{11} and C_{12} between two consecutive iterations is less than or equal to a chosen tolerance (Tol). The tolerance adopted in this paper is 10^{-5} .

This algorithm was implemented in a program (CalCxS) developed to obtain the damage curve from laboratory test results or computer simulations of fatigue tests. In this program, the value of the initial damage parameter (S_0), constant with each attempt, is arbitrated by the user. Therefore, it is recommended to run the program a few times with different values of S_0 . It is up to the user to choose the most appropriate curve according to the value of R^2 obtained in each attempt. This aspect is not a major issue, as good results are obtained in a few attempts.

3.3. Adjustment ($C = 1 - C_{11} S^{C_{12}}$) curve with the parameters C_{11} and C_{12}

The parameters C_{11} and C_{12} of the curve represented by Equation (9) that best fit the set of ordered pairs on iteration k , $(C_n^{(k)}, S_n^{(k)})$, are calculated using the Least Squares Method. Equation (9) can be written as:

$$(1 - C) = C_{11} S^{C_{12}} \quad (14)$$

Taking the natural logarithm on each side of Equation (14) yields:

$$\ln(1 - C) = \ln(C_{11}) + C_{12} \ln(S) \quad (15)$$

Equation (15) resembles the equation of a straight line, $y = a_0 + a_1 x$, where:

$$y = \ln(1 - C) \quad (16)$$

$$a_0 = \ln(C_{11}) \quad (17)$$

$$a_1 = C_{12} \quad (18)$$

$$x = \ln(S) \quad (19)$$

Finally, after obtaining the coefficients a_0 and a_1 of the least squares regression line, the new estimations of the damage curve parameters are computed as follows:

$$C_{11} = e^{a_0} \quad (20)$$

$$C_{12} = a_1 \quad (21)$$

4. RESULTS

To verify the proposed algorithm, fatigue tests with controlled sinusoidal strain were simulated using an axisymmetric model for three mixtures whose Prony series and the parameters of the vs. curve are known (Souza, 2018). Among the results of these simulations are the material integrities on the peak of each periodic action cycle. These integrities (C) with the characteristics of the strain fatigue test simulation (frequency, amplitude, duration) were used to obtain the referred parameters of the C vs. S curve, with the presented algorithm.

The fatigue test was simulated using the AEDCISO program (Almeida, 2019), which uses the S-VECD theory (Underwood et al., 2009). The AEDCISO program can simulate the fatigue test considering the stiffness degradation during the evolution of material damage. However, in the simulations performed for this article, this effect was not considered in order not to change the characteristics of the C vs. S original curve. As this is not the purpose of this article, no further

details on the methodology used in the AEDCISO program is presented. The parameters of the C vs. S curve were obtained with CalcCxS program (Almeida, 2019).

The constants of the Prony series of the mixtures are shown in Table 1. The α values with the C vs. S curve parameters are presented in Table 2 (Souza, 2018). In all simulations of fatigue test it was used the initial damage parameters, $S_0 = 100$. In each mixture, the frequencies 0.1; 0.5; 1.0; 5.0; 10.0 and 25.0Hz were considered with duration of 50000, 10000, 5000, 1000, 500 and 200 seconds, respectively, producing 5000 sine deformation cycles with amplitude (peak to peak), $\varepsilon_0 = 200 \times 10^{-6}$. In the laboratory, the tests must be performed with small strains that do not produce immediate failure in the specimens. In the case of computer simulation, this is not necessary. Thus, it was opted for strain amplitude value that produced the damage in a relatively small number of cycles.

The relative tolerance of 10^{-5} was considered in the algorithm stop criterion for obtaining parameters C_{11} and C_{12} of the C vs. S curve. To demonstrate the ability of the algorithm to find the parameters of the C vs. S curve, initial approaches to C_{11} and C_{12} were arbitrated very different from their known values. It was adopted $C_{11} = C_{12} = 0.01$, but other values (within the range 0 to 1) can be used. The values of C_{11} and C_{12} depend on S_0 and α associated with the C vs. S curve, as well as the stress units used in the test or in the simulation. To compare the resulting curves with the original curves, it was used in CalcCxS program, the same α values and initial damage values (S_0) close to those used in the fatigue tests simulations performed with the AEDCISO program. This is not always possible as they eventually produce C_{11} and C_{12} results that generate negative R^2 values. Thus, other values of S_0 were adopted to produce R^2 values close to 1.0.

Table 1 – Relaxation modulus (kPa) – Source: Souza (2018)

i	Asphalt Mixtures			ρ_i
	A	B	C	
	E_i	E_i	E_i	
1	3.83×10^2	4.18×10^2	2.98×10^2	2.0×10^{10}
2	4.13×10^2	4.02×10^2	3.17×10^2	2.0×10^9
3	8.63×10^2	7.93×10^2	6.58×10^2	2.0×10^8
4	1.83×10^3	1.59×10^3	1.38×10^3	2.0×10^7
5	3.96×10^3	3.23×10^3	2.93×10^3	2.0×10^6
6	8.96×10^3	6.86×10^3	6.43×10^3	2.0×10^5
7	2.21×10^4	1.56×10^4	1.50×10^4	2.0×10^4
8	6.24×10^4	3.96×10^4	3.90×10^4	2.0×10^3
9	2.13×10^5	1.17×10^5	1.19×10^5	2.0×10^2
10	8.30×10^5	4.02×10^5	4.25×10^5	2.0×10^1
11	2.92×10^6	1.40×10^6	1.55×10^6	2.0×10^0
12	6.02×10^6	3.70×10^6	3.98×10^6	2.0×10^{-1}
13	7.67×10^6	5.78×10^6	5.92×10^6	2.0×10^{-2}
14	7.85×10^6	7.31×10^6	6.96×10^6	2.0×10^{-3}
15	6.48×10^6	7.18×10^6	6.34×10^6	2.0×10^{-4}
16	4.72×10^6	5.99×10^6	4.91×10^6	2.0×10^{-5}
17	3.17×10^6	4.45×10^6	3.42×10^6	2.0×10^{-6}
18	2.03×10^6	3.08×10^6	2.24×10^6	2.0×10^{-7}
19	1.26×10^6	2.04×10^6	1.41×10^6	2.0×10^{-8}
20	7.67×10^5	1.31×10^6	8.66×10^5	2.0×10^{-9}
21	1.07×10^6	2.00×10^6	1.21×10^6	2.0×10^{10}
	$E_\infty = 3.64 \times 10^4$	$E_\infty = 3.19 \times 10^4$	$E_\infty = 3.89 \times 10^4$	

Table 2 – Experimental parameters for the asphalt mixture (A, B and C). Source: Souza (2018)

ASPHALT MIXTURES	α	C_{11}	C_{12}
A	2.93	1.56×10^{-4}	7.42×10^{-1}
B	3.07	4.44×10^{-4}	6.39×10^{-1}
C	3.02	4.36×10^{-4}	6.39×10^{-1}

Tables 3, 4 and 5 show the parameters C_{11} and C_{12} obtained with CalcCxS program for mixtures A, B and C, respectively. The tables also show the R^2 values for the curves and the number of iterations of the algorithm until the result is reached. These tables show that the parameters do not exactly match the original parameters shown in Table 2. This is due to use the pseudo deformation calculated as the product of the dynamic modulus by the deformation in the simulation of the fatigue test performed with the AEDCISO program (according with S-VECD theory), whereas in the CalcCxS program the pseudo deformation is given by Equation (13). The C vs. S curves plotted with the parameters of Tables 3, 4 and 5 are shown respectively in Figures 3, 4 and 5, with the original curve of each mixture. For all mixtures, it is observed that the C vs. S curves obtained with the algorithm basically coincide with each other, regardless of the frequency, and the proposed algorithm reached the original curve.

Table 3 – Parameters obtained for mixture A using CalcCxS program

FREQUENCY (Hz)	S_0	α	C_{11}	C_{12}	R^2	ITERATIONS
0.1	35	2.93	1.67×10^{-4}	7.36×10^{-1}	0.9998	13
0.5	55	2.93	1.67×10^{-4}	7.36×10^{-1}	0.9998	13
1.0	70	2.93	1.71×10^{-4}	7.34×10^{-1}	0.9996	13
5.0	85	2.93	1.72×10^{-4}	7.33×10^{-1}	0.9996	12
10.0	95	2.93	1.77×10^{-4}	7.31×10^{-1}	0.9994	14
25.0	95	2.93	1.74×10^{-4}	7.32×10^{-1}	0.9995	14

Table 4 – Parameters obtained for mixture B using CalcCxS program

FREQUENCY (Hz)	S_0	α	C_{11}	C_{12}	R^2	ITERATIONS
0.1	15	3.07	5.17×10^{-4}	6.20×10^{-1}	0.9982	9
0.5	30	3.07	4.72×10^{-4}	6.33×10^{-1}	0.9998	8
1.0	40	3.07	4.73×10^{-4}	6.33×10^{-1}	0.9998	11
5.0	55	3.07	4.73×10^{-4}	6.33×10^{-1}	0.9997	11
10.0	60	3.07	4.89×10^{-4}	6.30×10^{-1}	0.9995	8
25.0	65	3.07	4.84×10^{-4}	6.30×10^{-1}	0.9995	8

Table 5 – Parameters obtained for mixture C using CalcCxS program. Source: Authors (2019).

FREQUENCY (Hz)	S_0	α	C_{11}	C_{12}	R^2	ITERATIONS
0.1	20	3.02	4.92×10^{-4}	6.25×10^{-1}	0.9998	9
0.5	35	3.02	4.62×10^{-4}	6.33×10^{-1}	0.9998	10
1.0	45	3.02	4.66×10^{-4}	6.33×10^{-1}	0.9997	10
5.0	60	3.02	4.67×10^{-4}	6.32×10^{-1}	0.9997	12
10.0	65	3.02	4.82×10^{-4}	6.29×10^{-1}	0.9994	8
25.0	70	3.02	4.78×10^{-4}	6.30×10^{-1}	0.9995	9

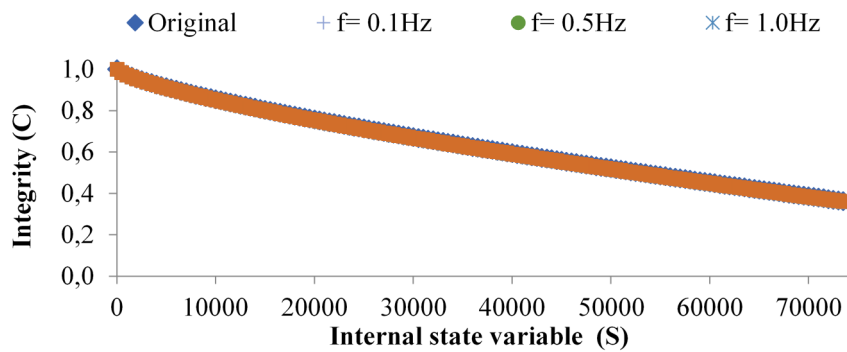


Figure 4. C vs. S curves for mixture A

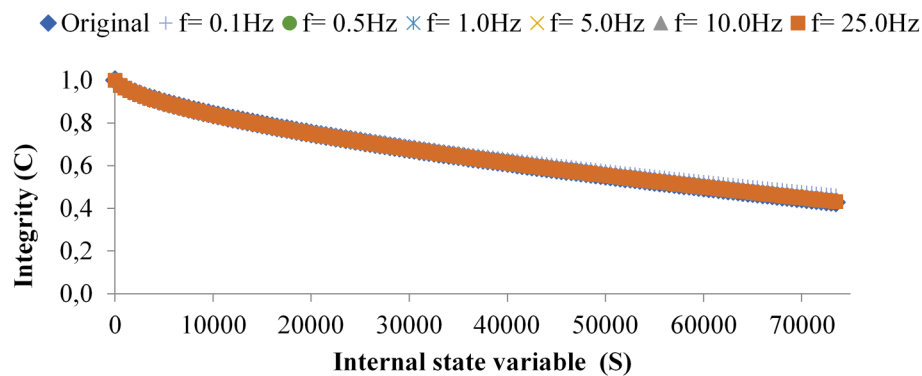


Figure 5. C vs. S curves for mixture B

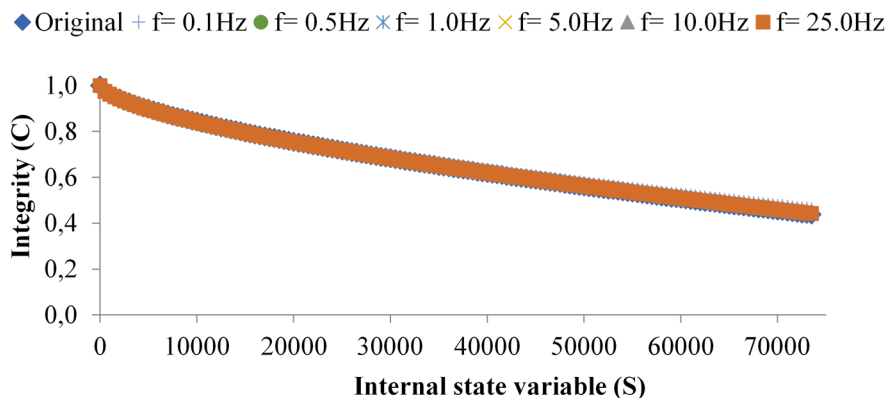


Figure 6. C vs. S curves for mixture C

5. CONCLUSIONS

This paper presents an algorithm to determine the damage characteristic curve of asphalt mixtures. Results demonstrate the efficiency of the algorithm proposed, since the parameters obtained for different frequencies produce approximately equal C vs. S curves. The algorithm converges to the damage curve parameters, even though the initial approximations of C_{11} and C_{12} are different from their original values, with R^2 values close to 1.0. All values of the parameter S_0 used in the algorithm are lower than the value of S_0 used in the fatigue test simulations. It was tried, in the CalcCxS program, the use of S_0 values equal to the value used in the simulations of fatigue tests, but it produced overflow during the running of the program. In this case, the

program issues a message requesting another S_0 value. Even so, the curves obtained by the algorithm adequately represent the original curves. The calculation of the pseudo strains with the use of the LVE theoretical responses in stress for a strain controlled action has the advantage to facilitate subsequent treatment of the data to obtain the C vs. S curve.

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REFERENCES

- ALMEIDA, F. J. P. (2019) Modelagem computacional de misturas asfálticas com a utilização do método dos elementos finitos, da teoria da viscoelasticidade linear e da mecânica do dano contínuo. 2019. 141 f. PhD Thesis, UFC-Brazil. <http://www.repositorio.ufc.br/handle/riufc/40405> (In Portuguese language).
- BABADOPULOS, L. F. A. L., FERREIRA, J. L. S., NASCIMENTO, L. A. H., SOARES, J. B., CASTELO BRANCO, V. T. F. (2016) Aging-effect incorporation into the fatigue-damage modelling of asphalt mixtures using the S-VECD Model. [https://doi.org/10.1061/\(ASCE\)MT.1943-5533.0001676](https://doi.org/10.1061/(ASCE)MT.1943-5533.0001676)
- FERRY, Y. D. (1980) Viscoelastic properties of polymers. John Wiley & Sons, Inc, New York (1980). ISBN: 978-0-471-04894-7 J. Eng. Mech-ASCE, 116 (4) (1990), pp. 751-772. [https://doi.org/10.1061/\(ASCE\)0733-9399\(1990\)116:4\(751\)](https://doi.org/10.1061/(ASCE)0733-9399(1990)116:4(751))
- KIM, Y. R. (2009) Modelling of asphalt concrete. ASCE Press and McGrawHill. ISBN: 9780071464628.
- KIM, Y.R. and LITTLE, D. N. (1990) One-dimensional constitutive modeling of asphalt concrete. Journal of Engineering Mechanics. Volume 116. Issue 4, April 1990. [https://doi.org/10.1061/\(ASCE\)0733-9399\(1990\)116:4\(751\)](https://doi.org/10.1061/(ASCE)0733-9399(1990)116:4(751))
- LAMBORN, M. J. and SCHAPERLY, R. A (1993) An investigation of the existence of a work potential for fiber-reinforced plastic. Journal of Composite Materials. Volume 27. Issue: 4, page(s): 352-382. Issue published: April 1, 1993. <https://doi.org/10.1177%2F002199839302700402>
- LEMAITRE, J. (1996) A course on damage mechanics. ISBN 978-3-642-18255-6. <https://doi.org/10.1007/978-3-642-18255-6>
- NASCIMENTO, L.A.H. (2015) Implementation and validation of the viscoelastic continuum damage theory for asphalt mixture and pavement analysis in Brazil. PhD Thesis, North Carolina State University, Raleigh, North Carolina.
- PAGEN, C. A. (1965) Rheological response of bituminous concrete. In Highway Research Record 67, HRB, National Research Council, Washington, D.C., 1965, pp. 1-26.
- PARK, S.W.; KIM, Y. R.; SCHAPERLY, R.A. (1996) A viscoelastic continuum damage model and its application to uniaxial behavior of asphalt concrete. Mechanics of Material, V. 24, pp. 241-255. [https://doi.org/10.1016/S0167-6636\(96\)00042-7](https://doi.org/10.1016/S0167-6636(96)00042-7)
- SCHAPERLY, R. A. (1981) On viscoelastic deformation and failure behavior of composite materials with distributed flaws. Advances in Aerospace Structures and Materials - AD-01, ASME, New York, pp. 5-20.
- SCHAPERLY, R. A. (1994) Nonlinear viscoelastic constitutive equations for composite materials based on work potentials. Proceedings 12th congress of applied mechanics (ed. Kobayashi), Appl. Mech. Rev., 47. <https://doi.org/10.1115/1.3124421>
- SCHAPERLY, R.A. (1984) Correspondence principles and a generalized j integral for large deformation and fracture analysis of viscoelastic media. Int. J. Fracture., 25, 195-223. <https://doi.org/10.1007/BF01140837>
- SOUZA, R. C. (2018) Avaliação do dano por fadiga em misturas asfálticas finas através de ensaios dinâmicos uniaxiais. Master Thesis. PUC-Brazil (in Portuguese language).
- UNDERWOOD, B., KIM, Y., SAVADATTI, S., THIRUNAVUKKARASU, S., GUDDATI, M. (2009) Response and fatigue performance modeling of ALF pavements using 3-d finite element analysis and a simplified viscoelastic continuum damage model. In: Asphalt Paving Technology: Association of Asphalt Paving Technologists-Proceedings of the Technical Sessions. [S.l.: s.n.], 2009. v. 78, p. 829-868.