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Mathematical model and math-heuristic for the aircraft recovery problem

Modelo e heurística matemática híbrida para o problema da recuperação de malha aérea

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ABSTRACT
The aircraft recovery problem (ARP) arises when unexpected events such as storms, airport closures, and unscheduled aircraft maintenance cause delays and/or flight cancellations, dismantling the original aircraft schedule. This work initially presents a mathematical model for the recovery of an airline schedule. Given the NP-Hard nature of the problem, the mathematical model cannot solve large instances. It has led to the development of a two-part math-heuristic: a mixed-integer programming network flow model to obtain a new schedule with minimum flight cancellations and delays; and an integer linear programming model to minimize flight aircraft exchanges from the original schedule. Applications of the heuristic to instances with up to 470 flights presented, in less than one minute of processing, results differing less than 0.5% from the optimal results, which allows to conclude that the heuristic qualifies for application to real cases of considerable size.

RESUMO
O problema de recuperação de malha aérea surge quando eventos inesperados como tempestades, fechamento de aeroportos e manutenção não programada de aeronaves provocam atrasos e/ou cancelamento de voos, inviabilizando o cumprimento da programação original da aeronave. Este trabalho inicia pela apresentação de um modelo matemático para a recuperação da programação de uma empresa aérea. Devido à natureza *NP-Hard* do problema, o modelo matemático não é capaz de resolver grandes instâncias. Tal circunstância levou ao desenvolvimento de uma heurística matemática composta por dois modelos: um modelo de fluxo em rede com programação inteira mista para gerar uma nova programação com o menor número de cancelamentos e atrasos de voos; e um modelo de programação linear inteira para minimizar as trocas de aeronaves associadas aos voos com relação à programação original. São apresentadas aplicações da heurística a instâncias com até 470 voos, para as quais se obtém, em menos de um minuto de processamento, soluções que distam menos de 0,5% das soluções ótimas, o que permite concluir que a heurística se qualifica para aplicações em casos reais de magnitude considerável.

1. INTRODUCTION

The aim of the aircraft recovery problem is to find a new schedule, i.e. to define flight delays or cancellations in face of disruption conditions.

To this end, a mathematical model was initially developed, but the NP-hard characteristic of the problem allowed only the solution of small instances. Therefore, a mathematical (math) heuristic was proposed to solve larger instances of schedule recovery.

Two objectives were pursued: (1) to minimize flight deviations; and (2) to re-establish the original flight schedule after a specified period, looking for a new schedule of flights with the lowest cost to the airline and with the least possible discomfort to passengers. The modeled disruptions were delays, cancellations and unscheduled maintenance. General costs were attributed to change in the original flight schedule. Transfer flights (ferry) were not considered.

2. LITERATURE REVIEW

According to Clausen *et al*. (2010), Teodorovic e Guberinic (1984) presented a heuristic to find a new flight schedule, affected by the unscheduled maintenance of an aircraft. Hu *et al*. (2015) demonstrated that the recovery problem is NP-Hard when deviations from the original schedule must be minimized. Still, mixed-integer programming network flow approaches dominate problem modeling, often using set decomposition or partitioning schemes (Yan & Yang, 1996; Yan & Young, 1996). Heuristics and metaheuristics have also been used, especially in integrated problems (Arguello, Bard & Yu, 1997), (Anderson, 2006).

Since the 2000s, alternative methods have been proposed for the recovery of schedules with more than 400 flights. However, desirable computational times of a maximum of 30 min (Serrano & Kazdab, 2017) have not been achieved for large instances. Another point is the treatment of maintenance situations. The literature is vast in maintenance planning, but scarce in the treatment of maintenance in operational framework.

Thengvall *et al.* (2001) proposed a multi-commodity mixed-integer programming network flow model to treat cancellations, delays and transfer flights at the same time. In addition, to allow flight aircraft exchanges - swap - for the same aircraft configuration. They have got results for large instances at acceptable execution times. Unscheduled maintenance was not covered; the authors pointed out difficulties in their treatment. Two suggestions were made: to treat the aircraft under maintenance as a separate commodity, or to treat the routes of aircraft under maintenance in a heuristic way.

Zhang *et al* (2016) presented a math-heuristic, combining network flow models with heuristics, with the objective of solving, in an integrated way, aircraft and passenger recovery problems. It was proposed to decouple the problem in two schedule planning problems: fleet assignment and aircraft rotation. A random aircraft rotation was considered, even without any optimization feature. An important consequence was that the maintenance arcs were not assigned to specific aircraft, leading to incorrect aircraft in maintenance.

No proposal adequately covering the impact of unscheduled maintenance was found in the literature, as well as able to solve the ARP in desirable computational time.

The present proposal to the aircraft recovery problem was directed towards overcoming these gaps, contemplating unscheduled maintenance and seeking computational solutions within adequate processing times to be used in business practice.

3. CHARACTERIZATION OF THE PROBLEM

The new schedule should be optimized, which means minimizing changes in flights, quantified by general costs. In addition, flight schedule recovery must be enforced, i.e. it must return to its original state after a time period called recovery period. The new flight schedule shall respect the airport capacity.

The flight schedule, in operational framework, is the set of flights whose seats have been sold and which must operate according to dates, times and types of aircraft agreed at the time of sale (Belobaba *et al*, 2009). The simplest way to represent it is by a list of flights, along with the rotation of aircraft. This is the specification of the date and on which specific aircraft each flight on the list will operate. Aircraft type or configuration is the model of the aircraft associated with a configuration of seats available for sale. Airbus 320 with 174 seats and Boeing 737 with 184 seats are examples.

Figure 1 presents an example of flight scheduling. Flights are represented by a flight number, a pair of airports, take-off and landing times, takeoff date and specific aircraft.

501 GRU BSB 12:00 13:30	501 07/01/2006 A320#1
504 BSB GRU 15:30 17:00	504 07/01/2006 A320#1
701 BSB SSA 12:00 14:30	701 07/01/2006 A320#2
706 SSA BSB 9:00 11:00	706 07/01/2006 A320#2
(a) Flight List	**(b) Aircraft Rotation**

Figure 1. Flight list and rotation of A320 aircraft with174 seats

The flight schedule is also usually presented by a Gantt chart.

The schedule must comply with two rules when constructing:

1. An aircraft needs a minimum time on the ground - turnaround time - between a landing and the subsequent takeoff (Clarke *et al*. ,1997);

2 In an airport, the number of landings and takeoffs in an hour time band is limited (ANAC Resolution 440/2017). The limits refer to the operational capacity of the airport and are disclosed by the airport manager to the airline. The available slots – time of flight - are assigned to companies respecting these limits.

3.1. Space-time network

The mathematical representation of the flight schedule used is the Space-Time Network. Landing and takeoff events are arranged on a timeline for an airport and type of aircraft. Nodes correspond to a time at an airport and derive from landing and takeoff events. These are called intermediate nodes. The arcs correspond to the flights, which are the activities that connect two nodes at different airports. Two consecutive nodes in an airport are connected by a ground arc, activity representing aircraft on ground or maintenance event (Clausen *et al*, 2009).

The position of the node in the timeline refers to the time of the event. This time consists of the sum of the time and date of the event converted into minutes.

For constructive reasons, three other types of nodes are added: 1 - aircraft entry nodes, which are the points where aircraft begin their journeys; 2 - exit nodes, points ere planes finish their journeys; 3 – recovery nodes, points from which flight changes are not allowed due to disruption. The characteristics of a node are: time, location, type of aircraft, list of flight arcs that arrive at the node, list of flight arcs that leave the node, arrival ground arc and exit ground arc. Figure 2 shows the space-time network assembled from the flights in Figure 1.

The network, as shown in Figure 2, joins all events operated by one type of aircraft or commodity – A320 in the example. The complete network is the combination of all different networks, called the multi-commodity space time network. Flights can swap from original aircraft only within the aircraft type, or commodity.

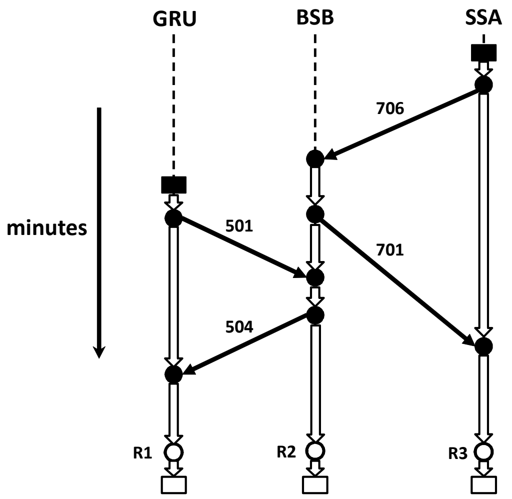


Figure 2. Network Space-Time aircraft type A320 with 174 seats

3.2. Disruptions and changes

Disruptions are external events that affect flight scheduling, causing flight delays or cancellations. Delay is the amount of time a flight remains on the ground after the originally planned time. In Brazil, ANAC, for the purpose of slot allocation, considers a flight to be delayed only after 15 minutes, according to its Resolution 338 of 2014. In this work, it is considered that the delay in departure implies delay in the landing of the same flight. Cancellation is the complete interruption of service; the flight just ceases to happen.

Unscheduled Maintenance is a period in which an aircraft cannot fly and waits on the ground due to some failure. Reduction of airport capacity is the definition of new limits on the number of landings and takeoffs possible at a given time due to operational or climatic reasons.

Flights undergo different changes due to the disruptions. Change of aircraft, delays and cancelations are allowed in this work.

4. MATHEMATICAL MODELING

A multi-commodity network flow model, in which each type of aircraft is a commodity, has been proposed to find the aircraft recovery problem solution. A space-time network was set up for each type of aircraft, where each specific aircraft of that type operates its flights, represented by arcs. The objective is to minimize total cost, quantified by each schedule change. The restrictions impose disruptions to the flight schedule, ensuring coverage of flights (each flight operated once or cancelled), balancing the network, correctly assigning airport capacity and returning to normal operations. The mathematical model is described as Model 1, below.

Datasets:

F: *flights present in the initial flight schedule*

N_INT: *intermediate nodes;*

N_ENT: *aircraft entry nodes;*

K_{fc}: *specific aircraft of the same type as the one carrying out the flight which gave rise to the fc delay option in the initial schedule;*

K_n: *specific aircraft of the same type as the node n represents;*

FC_f: *arcs representing flight delay options f;*

FC_IN_n: *arcs that reach node n;*

FC_OUT_n: *arcs coming out of node n;*

GA_IN_n: *ground arches arriving at node n;*

GA_OUT_n: *ground arches coming out of node n;*

$AERPS$: *airports present in the initial flight schedule;*

TM: *type of movements: landing or taking off;*

FH: *all hour time bands presented in the space-time network;*

$FC_{aep,tm,fh}$: *flight arcs that give rise to events of a type of tm movement at the airport aep, whose event times are within an fh hour time band;*

K_m: *specific aircraft undergoing some preventive or unscheduled maintenance;*

FC_M_k: *maintenance arcs for aircraft k;*

F_{delay}: *flights suffering from a delay disruption;*

FC_DELAY_f: *arcs representing delay options that need to be disabled to ensure the delay disruption imposed at f;*

F_{cancel}: *flights suffering from a cancellation disruption*

N_OUT: *Nodes that mark the end of the recovery period.*

Parameters:

$cost\_dly\_swp_{fc,k}$: *cost of delay and change of aircraft of flight arc* $x_{fc,k}$*;*

$cost\_cancel_f$: *cancelation cost of flight f;*

$Cap_{aep,tm,fh}$: *limit hour capacity by type of operation tm (landing or taking off), by time band fh and by airport aep;*

$N\_INPUT_{n,k}$: *binary parameter that set if the specific aircraft k enters in the network in n node;*

p_n: *number of aircrafts positioned in exit ground arc of recovery node n.*

Decision variables:

$x_{fc,k}$: *binary decision variable indicates if flight or maintenance arc fc by aircraft k operates;*

y_f: *binary decision variable indicates if flight f is canceled;*

$z_{ga,k}$: *binary decision variable indicates if aircraft k is in ground arc ga (arriving or exiting) nodes.*

Model 1: Exact Integrated Mathematical Model for the Aircraft Recovery Problem:

$$\min \sum_{\substack{fc \in FC_f \\ f \in F}} \sum_{k \in K_{fc}} x_{fc,k} * cost\_dly\_swp_{fc,k} + \sum_{f \in F} y_f * cost\_cancel_f \quad (1)$$

Subject to:

$$\sum_{fc \in FC_f} \sum_{k \in K_{fc}} x_{fc,k} + y_f = 1 \qquad \forall f \in F \quad (2)$$

$$\sum_{fc \in FC_{IN_n}} x_{fc,k} + \sum_{ga \in GA_{IN_n}} z_{ga,k} = \sum_{fc \in FC_{OUT_n}} x_{fc,k} + \sum_{ga \in GA_{OUT_n}} z_{ga,k} \qquad \forall n \in N\_INT, \forall k \in K_n \quad (3)$$

$$\sum_{ga \in GA\_OUT_n} z_{ga,k} = N\_INPUT_{n,k} \qquad \forall n \in N\_ENT, \forall k \in K_n \quad (4)$$

$$\sum_{fc \in FC_{aep,tm,fh}} \sum_{k \in K_{fc}} x_{fc,k} \leq Cap_{aep,tm,fh} \qquad \forall aep \in AERPS, \forall tm \in TM, \forall fh \in FH \quad (5)$$

$$\sum_{fc \in FC\_M_k} x_{fc,k} = 1 \qquad \forall k \in K_m \quad (6)$$

$$\sum_{fc \in FC\_DELAY_f} \sum_{k \in K_{fc}} x_{fc,k} = 0 \qquad \forall f \in F_{delay} \quad (7)$$

$$y_f = 1 \qquad \forall f \in F_{cancel} \quad (8)$$

$$\sum_{ga \in GA\_OUT_n} \sum_{k \in K_n} z_{ga,k} = p_n \qquad \forall n \in N\_OUT \quad (9)$$

It should be noted that in equations (4) and (9), the left sides contain sums of binary variables, the results of which are natural numbers.

The objective function (Equation 1) is composed of two terms: the first is the cost of the flight $x_{fc,k}$, sum of the cost of the delay with the cost of changing aircraft; the second is the cost of cancelling a flight. Therefore, the minimization of the objective function seeks a schedule of flights that is as close as possible to the initial schedule.

Constraint (Equation 2) is the flight coverage. It indicates that a flight operates on any of the delay options for any of the aircraft of its aircraft type, or is cancelled. Balancing, always present in network flow models, is represented by the constraint (Equation 3). Equation 4 marks the entry of specific aircraft into the network by the input nodes, one by one.

Airport runway capacity limits are guaranteed by constraint (Equation 5). Flight events in an hour time band must respect these limits. This constraint couples all commodities of the model. Aircraft on the ground does not consume capacity; therefore, ground and maintenance arcs are not accounted for.

Maintenance, unscheduled or not, is guaranteed by constraint (Equation 6).

Delays imposed by disruptions are governed by constraint (Equation 7), which guarantees a minimum delay equal to that requested. A cancellation imposed by disruption is made by defining as 1 the decision variable y_f corresponding to the flight, as shown in restriction (Equation 8).

Finally, the constraint (Equation 9) is the one that effectively ensures that the recovery is resolved by the end of the requested recovery period. To do so, a number of aircraft p_n coming out of the recovery nodes are imposed.

5. MATHEMATICAL HEURISTICS

The proposed heuristic method to solve the aircraft recovery problem decouple the mathematical model in two parts: the first, called Fleet Assignment, defines how much each flight will be delayed, or whether it will be cancelled respecting the number of aircraft in the fleet and airport capacity; the second defines the Aircraft Rotation, that is, which specific aircraft will perform each flight. This uncoupling hinders the optimization of the problem, but it is capable of treating large and very disrupted instances, as shown by Thengvall *et al* (2001).

Fleet assignment and aircraft rotation are resolved using network flow models, the first being fleet assignment, a multi-commodity mixed-integer linear programming model integrating all types of aircraft subject to airport capacity.

5.1. Fleet Assignment

Fleet assignment is derived from the Exact Integrated Mathematical Model – Model 1, applying the simplifying hypothesis that all aircraft in a configuration are identical. This hypothesis is valid because within a network-time space, all aircraft belong to the same type and any flight can be executed by any aircraft in business practice. Only maintenance arcs were designed to be operated by specific aircraft. It was found, empirically, that these arcs represent about 2% of the flight schedule of a given type of aircraft. Therefore, the use of fleet allocation remains valid in this stage of heuristic, as best explained below.

The above hypothesis is verified because the fleet assignment does not choose the specific aircraft; this is done in the subsequent stage, Aircraft Rotation. Fleet assignment points out which arcs should compose the solution, i.e. it decides whether flights are cancelled or operate and with which delay options. Maintenance flights are treated as a hard restriction, meaning they need to occur without delay. As far as no specific aircraft is chosen, minimum cost solution could operate with a change of aircraft of a maintenance arc, what leads to an infeasible Aircraft Rotation solution. If that happens, it is corrected later to achieve the final solution.

The mathematical model of the assignment is represented in Model 2. It is basically the same as the exact integrated model, removing the aircraft choice from decision variables and arcs. The aircraft enters the Space-Time Network, not one by one, as verified in the constraint (Equation 4). Consequently, the ground variables now represent multiple aircrafts on the ground, which transforms the Fleet Assignment into a mixed-integer flow problem. In this type of problem, some decision variables are integers, rather than binaries (Bradley *et al*, 1977).

The same definitions of datasets, parameters, and decision variables described in section 4 were considered, with the following changes.

Datasets:

FC_M: *preventive or unscheduled maintenance arcs;*

FC_M_f: *maintenance arcs for the different airports where unscheduled maintenance f may occur.*

Parameters:

$cost\_dly_{fc}$: *cost of delay of an xfc flight arc;*

N_INPUT_n: *Number of aircraft entering the network by input node n.*

Decision Variables:

x_{fc}: *Indicates whether or not the fc flight or maintenance arc is operated by any aircraft, binary;*

z_{ga}: *Indicates the number of aircraft waiting in the ga ground arc.*

Model 2: Mathematical Model Fleet Assignment.

$$min \sum_{\substack{fc\in FC_f \\ f\in F}} x_{fc} * cost\_dly_{fc} + \sum_{f\in F} y_f * cost\_cancel_f \tag{10}$$

Subject to:

$$\sum_{fc\in FC_f} x_{fc} + y_f = 1 \quad \forall f \in F \tag{11}$$

$$\sum_{fc\in FC\_IN_n} x_{fc} + \sum_{ga\in GA\_IN_n} z_{ga} = \sum_{fc\in FC\_OUT_n} x_{fc} + \sum_{ga\in GA\_OUT_n} z_{ga} \quad \forall n \in N\_INT \quad (12)$$

$$\sum_{ga\in GA\_OUT_n} z_{ga} = N\_INPUT_n \quad \forall n \in N\_ENT \quad (13)$$

$$\sum_{fc\in FC_{aep,tm,fh}} x_{fc} \leq Cap_{aep,tm,fh} \quad \forall aep \in AERPS, \forall tm \in TM, \forall fh \in FH \quad (14)$$

$$\sum_{fc\in FC\_M_f} x_{fc} = 1 \quad \forall f \in FC_M \quad (15)$$

$$\sum_{fc\in FC\_DELAY_f} x_{fc} = 0 \quad \forall f \in F_{delay} \quad (16)$$

$$y_f = 1 \quad \forall f \in F_{cancel} \quad (17)$$

$$\sum_{ga\in GA\_OUT_n} z_{ga} = p_n \quad \forall n \in N\_OUT \quad (18)$$

The interpretation of the equations of the assignment model is equivalent to that of the exact integrated mathematical model (Model 1). However, some changes stand out: the objective function (Equation 10) does not contain the cost of swapping aircraft; the ground arches keep the information of the amount of aircraft that are waiting there. This can be observed when aircraft enter the network by restriction (Equation 13); balancing (Equation 12), in this case, ensures that the number of aircraft that reaches a node must be equal to that coming out of it. The recovery condition (Equation 18) does not need to add the contributions of different aircraft, because this quantity is expressed in the decision variable z_{ga}.

5.2. Aircraft Rotation

The mathematical model of aircraft rotation aims to find the specific aircraft of each chosen arc in the fleet assignment solution. It is also a simplification of the exact integrated mathematical model – Model 1:only the constraints and portions of the objective function related to the choice of the specific aircraft remain.

The objective of the optimization attempt is to minimize differences in relation to the initial schedule, i.e. aircraft swaps. This differs from the proposal of Zhang *et al*. (2016), in which the rotation was randomly chosen. As there is no further change in flights than the definition of the specific aircraft, the use of the airport capacity is fixed. Thus, without the capacity restriction, there is no more competition among the different types of aircraft for the consumption of the resource. Therefore, the rotation can be solved for each type of aircraft separately.

The mathematical model is described in Model 3 below. The definition of the sets and decision variables are the same as shown in Model 1 - Exact Integrated Mathematical Model for the Aircraft Recovery Problem. However, the cost parameter present in the objective function relates only to the change of aircraft.

Model 3: Mathematical Model Aircraft Rotation

$$min \sum_{\substack{fc\in FC_f \\ f\in F}} \sum_{k\in K_{fc}} x_{fc,k} * cost\_swp_{fc,k} \quad (19)$$

Subject to:

$$\sum_{k\in K_{fc}} x_{fc,k} = 1 \quad \forall f \in F \quad (20)$$

$$\sum_{fc\in FC_{IN_n}} x_{fc,k} + \sum_{ga\in GA_{IN_n}} z_{ga,k} = \sum_{fc\in FC_{OUT_n}} x_{fc,k} + \sum_{ga\in GA_{OUT_n}} z_{ga,k} \quad \forall n \in N_{INT}, \forall k \in K_n \quad (21)$$

$$\sum_{ga\in GA\_OUT_n} z_{ga,k} = N\_INPUT_{n,k} \quad \forall n \in N\_ENT, \forall k \in K_n \quad (22)$$

$$\sum_{fc\in FC\_M_k} x_{fc,k} = 1 \quad \forall k \in K_m \quad (23)$$

The objective function (Equation 19) includes only the cost of the exchange; it is not necessary to include a coverage restriction on ground arcs, because the aircraft entry restriction (Equation 22) is in charge of differentiating these arcs.

The model ensures, through restriction (Equation 23), that the maintenance arcs are operated by the aircraft established in the initial schedule or by the disruption.

This model may not have a feasible solution. The infeasibilities appear precisely because the previous stage, fleet assignment, is not able to differentiate the aircraft, as explained previously. The interdependence of aircraft rotation in relation to aircraft types allows to isolate the configurations in which such infeasibilities occur, for later adjustment.

This possibility led to the proposition of a Hybrid Mathematical Heuristic to solve the identified infeasibilities.

6. HYBRID MATHEMATICAL HEURISTIC

The hybrid mathematical heuristic code for the aircraft recovery problem is shown below (Figure 3). In it, the Fleet Assignment is called, generating a partial result; then, Aircraft Rotation is performed iteratively for each aircraft type present in the initial schedule. The types of aircraft whose optimal results are achieved are stored as part 1 of the final solution. The aircraft types whose Aircraft Rotation solution is infeasible are brought together on a separated schedule. This schedule, part of the total, is solved through the Exact Integrated Mathematical Model - Model 1, which is able to find solutions without falling into the infeasibilities, once it solves fleet assignment and aircraft rotation simultaneously.

```
Assignment Result ← Fleet Assignment (Initial Schedule, Total airport capacity)
For each Aircraft configuration[J] of Initial Schedule Do
   Temp Rotation ← Aircraft Rotation (Assignment Result[J])
   If Temp Rotation is Feasible Then
        Result Part 1 ← Result Part 1 ∪ Temp Rotation
        ListConf_OK ← ListConf_OK ∪ { J }
   Else
        ListConf_Problem ← ListConf_Problem  ∪  { J }
   End If
End For
New system Capacity ← Total airport capacity – {Result_Part 1}
Final Rotation ← Result Part 1
Se ListConf_Prob ≠ ϕ Então
   Update Rotation ← Exact Model(Initial Schedule, ListConf_Problem, New system Capacity)
   Final Rotation ← Final Rotation  ∪ Update Rotation
Fim Se
Retorna Final Rotation
```

Figure 3. Pseudocode of the hybrid mathematical heuristic

It should be noted that the call to the exact integrated model by Hybrid Mathematical Heuristic is not intended to solve the aircraft recovery problem using the Exact Integrated Model 1, but only to solve a subpart of the schedule in which aircraft rotation has not found a feasible solution.

Care must be taken in terms of the available airport capacity delivered to the exact model. Certainly, it is not the total capacity initially considered, because part of it was consumed by the operations belonging to part 1 of the solution. Thus, it is necessary to deregister them from the total capacity before executing the exact model. This adjustment in airport capacity is made in step 11 of the heuristic. It ensures that the final solution of the aircraft recovery problem does not exceed the capacity of the airport. On the other hand, it can clearly lead to suboptimal solutions, because airport resources are being divided heuristically. Therefore, a validation of the method is essential.

7. APPLICATIONS AND RESULTS

7.1. Instances

The instances used to test the proposed method were those created by ROADEF for the operational research challenge in 2009 (http://www.roadef.org/challenge/2009/en/). They are subdivided into three groups: A, B and C, according to the size and level of disruption, that is, to the complexity of solution. Table 1 provides a description of the 31 instances divided by groups. For each instance, it shows the initial schedule in terms of the number of aircraft (Acft), flights (Flights), airports involved (Aerps) and types of aircraft or configuration (Confg).

The disruptions are presented in Table 1 subdivided by types: flight - number of flights that suffer disruption (Flights), total delay in minutes (Delay) and number of cancellations (CNL), airport - number of airports that suffer capacity reduction (Aerps) in a certain number of hour time bands (Hours) and aircraft - number of aircraft in unscheduled maintenance (Acft), totaling hours without receiving flights (Hours).

Table 1 - Description of the instance (ROADEF 2009)

						Perturbações						
								Voo	Aeroporto			Aeronave
Group	ID	Acft	Flights	Aerps	Confg	Voos	Delay (min)	CNL	Aerps	Hours	Acft	Hours
	1	81	464	35	15	63	2,670	-	-	-	-	-
	2	81	464	35	15	106	6,225	1	-	-	-	-
	3	81	464	35	15	79	5,550	4	-	-	1	15
	4	81	464	35	15	41	1,785	-	4	4	-	-
A	5	81	928	35	15	-	-	-	35	560	-	-
	6	81	464	35	15	63	2,670	-	-	-	-	-
	7	81	464	35	15	106	6,225	1	-	-	-	-
	8	81	464	35	15	79	5,550	4	-	-	1	15
	9	81	464	35	15	41	1,785	-	4	4	-	-
	10	81	928	35	15	-	-	-	35	560	-	-
	1	251	2,556	44	30	229	11190	-	-	-	-	-
	2	251	2,556	44	30	224	12,315	30	-	-	-	-
	3	251	2,556	44	30	228	11,115	-	-	-	1	29
	4	251	2,556	44	30	229	11,190	-	1	2	-	-
B	5	251	2,556	44	30	-	-	-	2	32	-	-
	6	251	2,556	44	30	228	11,190	1	-	-	-	-
	7	251	2,556	44	30	224	12,315	30	-	-	-	-
	8	251	2,556	44	30	228	11,115	-	-	-	1	29
	9	251	2,556	44	30	229	1,190	-	1	2	-	-
	1	614	6,102	168	30	-	-	-	1	8	1	39
	2	614	6,102	168	30	-	-	-	-	-	1	39
	3	614	6,102	168	30	-	-	-	1	7	1	28
	4	614	6,102	168	30	-	-	-	-	-	1	29
	5	81	464	35	15	78	5,520	4	-	-	3	42
C	6	81	928	35	15	-	-	-	35	560	3	114
	7	81	464	35	15	78	5,520	4	-	-	3	42
	8	81	928	35	15	-	-	-	35	560	3	114
	9	251	2,556	44	30	228	11,115	-	-	-	3	77
	10	251	2,556	44	30	-	-	-	2	32	1	36
	11	251	2,556	44	30	227	11,100	-	-	-	4	98
	12	251	2,556	44	30	-	-	-	2	32	3	95

Disruptions occur alone – as the case in Instance A1, where there are only flight disturbances, or in different combination – as in Instance C3, where airport capacity reduction and unscheduled maintenance occur simultaneously. Unscheduled maintenance has an average of 15 hours per aircraft in group A instances, which is more than half a day aircraft on ground. The delays provided on the instances start from 4 min, but due to network discretization, it was approximated to the higher multiple of 15. Thus, the model deals with a situation more disturbed than the real one, which can be seen as a conservative approach for the calculation of the final cost.

Mathematical models were solved using GUROBI 7.51 from Gurobi Optimization Inc. http://www.gurobi.com. Heuristics codes were written in C++ using Microsoft Visual Studio Community 2017 Version 15.4.0. The operating system was Windows 10 Enterprise, the processor was an Intel® Core ™ i7-7600U CPU @ 2.80GHz 2.90GHz and 8.00GB RAM.

7.2. Applications and Results of the Exact Model

The instances were initially solved exactly, using the mathematical model 1. The 8 instances of group A, which have 464 flights, were resolved by Gurobi in less than two minutes each one, reaching the optimal result. For all the other 23 instances remaining, Gurobi did not even get to linear relaxation before 2 hours of processing. The impossibility of solving the instances of groups B and C through the exact model within an acceptable time limit led to the application of the mathematical heuristic to solve large, disrupted instances.

7.3. Applications and Results of the Mathematical Heuristic

The parameters used in heuristic applications and in exact model were 10€ per minute delayed per flight and €20,000 per cancelled flight (Zhang *et al*, 2016).

These figures derive from cost estimates for the airline in the event of a flight change, which were empirically determined by ROADEF for the operational research challenge (http://www.roadef.org/challenge/2009/en/): 1€ for aircraft exchange, empirically determined to direct the algorithm, without effectively representing a new cost for the airline; 20 minutes as the maximum time of the whole process for each instance, that is the maximum period considered by airline operations centers to solve the recovery problem (PETERSEN *et al*, 2012); 50 delay options for all instances, except C1, C2, C3 and C4, which had 40 options (Zhang *et al*, 2016), due to the large number of flights, which would make computational time very high.

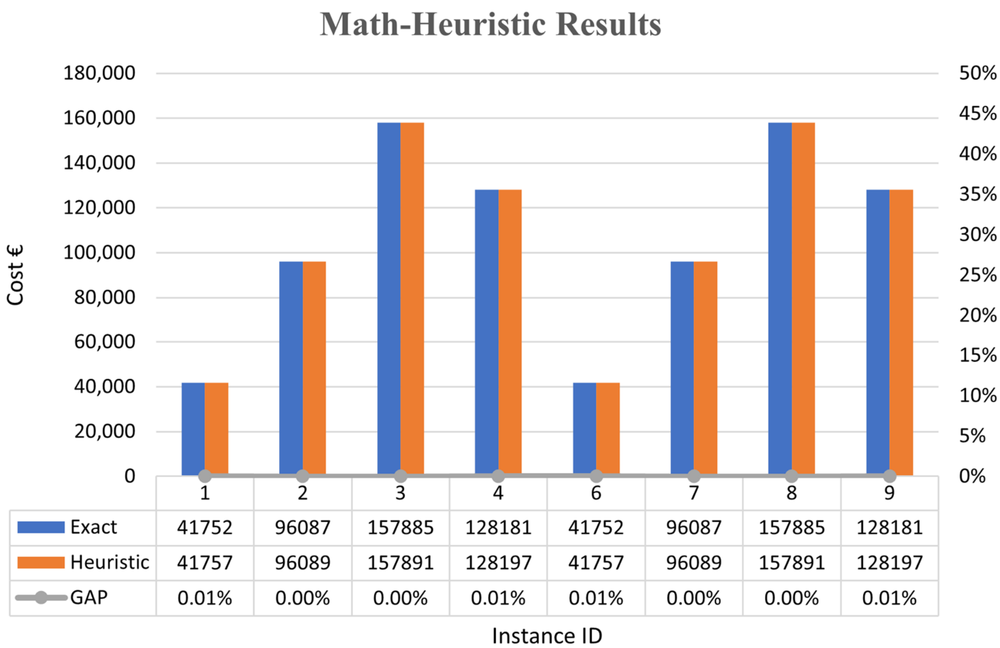
The same instances described in Table 1 of item 7.1 were considered.

7.3.1. Validation of the Mathematical Heuristic

Figure 4 shows the results of the application of the mathematical heuristic compared to optimal results found by the exact model for the same instances – group A.

Group A instances containing 464 flights were tested. In four of the eight instances, the difference was less than 0.001% and, in the other four, the difference was 0.01%. That is, the proposed heuristic produced results very close to the optimal results for the instances considered.

Differences in the objective function value are due to different aircraft rotation settings. The same cancellation and delay situation can be operated by different aircraft rotations.



	1	2	3	4	6	7	8	9
Exact	41752	96087	157885	128181	41752	96087	157885	128181
Heuristic	41757	96089	157891	128197	41757	96089	157891	128197
GAP	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%

Figure 4. Comparison of Exact Model and Heuristic Results – Group A

Despite the small differences in the objective function value, mathematical heuristics presented computational times substantially lower than the exact resolution of recovery, as shown in Figure 5.

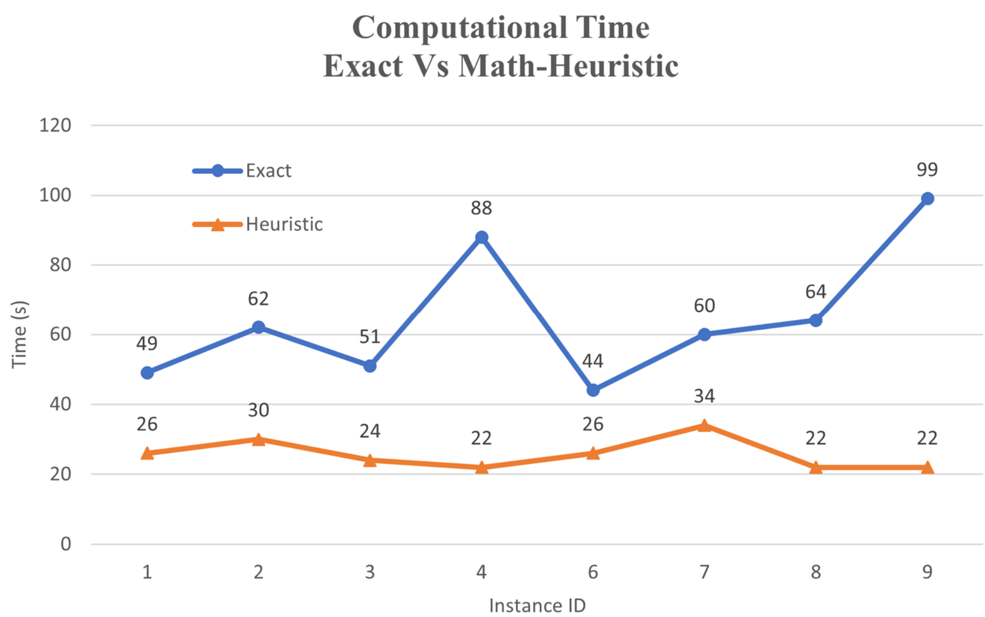


Figure 5. Execution times of Exact Model and Mathematical Heuristic – Group A

7.3.2. Hybrid Model Results

Table 2 contains the results of group A, B, and C instances resolved using the hybrid mathematical heuristic. The metrics presented are Regularity deviation (ΔReg), Punctuality deviation (ΔPunc), % of change of aircraft (% Swap), GAP_Atrib explained below, Total cost, Total time of execution, Final regularity (Reg = #Flights - #Cancelled Flights / #Flights), final punctuality 15 minutes and 60 minutes (P 15 = #Flown Flights - #Delayed flight (> 15min) / #Flown Flights).

No solutions were found for instances C 1 and C 2. Gurobi warned that the exact mathematical models for some configurations of these instances were unfeasible. This suggests that these instances may require ferry flights to be resolved, which were considered by ROADEF. As already mentioned, in the instances of groups B and C, the exact model was not able to find solutions at reasonable run time. For the purpose of analyzing the quality of these solutions, the GAP_Atrib was proposed. This represents the result of the heuristic excluding the number of aircraft swaps, compared to the value of the fleet assignment (step 1 of the heuristic – Figure 3).

7.3.3. Hybrid Model Runtime Analysis

The execution times of the hybrid model were less than 1 minute for type A instances with up to 500 flights, well below the maximum set parameter of 20 minutes (Petersen *et al*, 2012). Type B instances, with about 2,300 flights and type C instances with 6,000 flights, were executed in a period ranging from 10 to 17 minutes in most cases. This shows that the number of flights increases the processing time given the consequent increase in the calculation network, but not linearly.

Table 2 - Hybrid Math-Heuristic results

Grupo	IDD	ΔReg (pp)	ΔPunc(pp)	% Swap	GAP_Atrib	Cost	Time (s)	Reg	P15	P60
	1	0.0	-7.5	12.3%	0.0%	41,757	26	100.0%	78.9%	88.6%
	2	0.0	-4.1	8.4%	0.0%	96,089	30	99.8%	73.0%	86.2%
	3	0.0	-6.5	8.9%	0.0%	157,891	24	99.1%	76.3%	85.4%
	4	0.0	-36.6	20.9%	0.0%	128,197	22	100.0%	54.5%	71.3%
A	5	-2.4	-49.8	36.4%	0.0%	1,863,830	411	97.6%	50.2%	59.7%
	6	0.0	-7.5	12.3%	0.0%	41,757	26	100.0%	78.9%	88.6%
	7	0.0	-4.1	8.4%	0.0%	96,089	34	99.8%	73.0%	86.2%
	8	0.0	-6.5	8.9%	0.0%	157,891	22	99.1%	76.3%	85.4%
	9	0.0	-36.6	20.9%	0.0%	128,197	22	100.0%	54.5%	71.3%
	10	-2.4	-49.8	36.4%	0.0%	1,863,830	423	97.6%	50.2%	59.7%
	1	-0.5	-10.2	19.2%	0.2%	740,387	885	99.5%	80.8%	89.1%
	2	-1.2	-12.7	25.1%	0.0%	1,738,427	614	97.6%	78.4%	87.5%
	3	-0.6	-10.8	20.9%	5.0%	804,882	904	99.4%	80.3%	88.9%
	4	-0.7	-12.6	24.9%	1.5%	915,331	576	99.3%	78.4%	87.7%
B	5	-14.0		14.3%	0.0%	7,359,614	999	86.0%	92.4%	96.3%
	6	-0.5	-10.2	19.2%	0.2%	740,387	869	99.5%	80.8%	89.1%
	7	-1.2	-12.7	25.1%	0.0%	1,738,427	614	97.6%	78.4%	87.5%
	8	-0.6	-10.8	20.9%	4.9%	805,331	926	99.4%	80.2%	88.8%
	9	-0.7	-12.6	24.9%	1.5%	915,331	619	99.3%	78.4%	87.7%
	1	-	-	-	-	-	-	-	-	-
	2	-	-	-	-	-	-	-	-	-
	3	-0.2	-2.7	3.0%	0.0%	350,334	1.126	99.8%	97.3%	98.6%
	4	0.0	-0.3	1.4%	8.0%	44,800	10.211	99.1%	99.7%	99.9%
	5	0.0	-8.7	11.7%	0.0%	165,104	25	99.1%	74.3%	83.7%
C	6	-3.0	-49.6	35.8%	0.0%	1,929,522	638	97.0%	50.4%	61.2%
	7	0.0	-8.7	11.7%	0.0%	165,104	22	99.1%	74.3%	83.7%
	8	-3.0	-49.6	35.8%	0.0%	1,929,522	636	97.0%	50.4%	61.2%
	9	-0.7	-10.7	22.6%	0.3%	831,973	903	99.3%	80.4%	88.6%
	10	-13.5	-12.9	17.2%	0.0%	7,318,530	1.200	86.5%	87.1%	93.5%
	11	-0.7	-10.4	22.2%	0.3%	882,014	912	99.3%	80.8%	88.7%
	12	-13.5	-16.5	25.5%	0.1%	7,464,213	2.393	86.5%	83.5%	90.9%

The complexity of the disruption framework is a predominant factor. Instance B5 is marked by an important airport capacity reduction. In it, many infeasibilities appeared, which were solved with the exact model. This process was responsible for its long execution.

C4 and C12 instances required more than 20 minutes to be resolved, what disqualifies them for practical purposes. However, a deeper analysis showed that the C4 instance spent a lot of time trying to resolve the infeasibilities and, in the end, ended up canceling 2 flights in a row. Instance C12, much more complex in terms of disruption, was resolved in less time, but with more cancellations (high ΔReg). This shows that cancellation, although its high cost, is often the only solution to achieve the goal of re-establishing operations.

The results of the mathematical heuristic allow to obtain the composition of the execution times of each sub-step in the total time (Figure 6). Apart from instance B5, the average composition of the times was calculated as: 61% for Fleet Assignment, 24% for Aircraft Rotation, 11% for Exact method and 5% for other - pre- and post-processing processes. The computational cost-benefit of the call to the exact model is verified – step 14 of the Heuristics of Figure 3. The infeasibility of the entire system is solved by spending 11% of the computational resource. Fleet assignment takes longer (61%), as most schedule decisions are resolved at this stage.

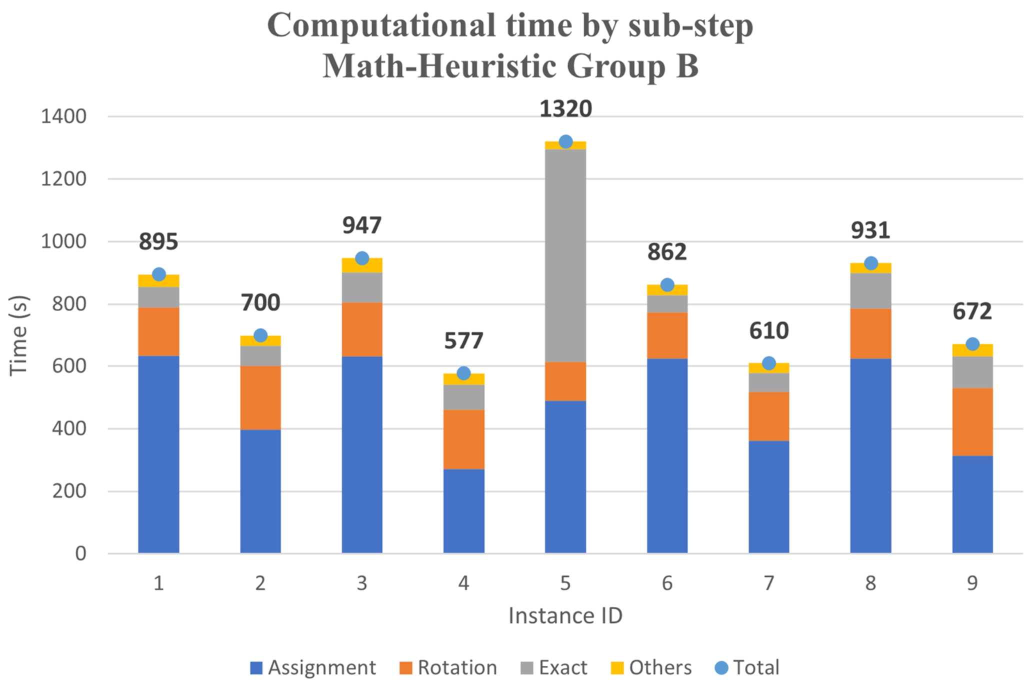


Figure 6. Times of sub-steps. Group B

7.3.4. Analysis of the relaxation of the recovery restriction

The recovery constraint (Equation 9 or Equation 18) marks the end of the recovery period and is considered a strong constraint. Flights are delayed and cancelled in order to meet this constraint.

Intuitively, the relaxation would entail two facts: schedule would not return to normal or the end of the recovery period is pushed forward. In order to better understand the effects of this relaxation, the recovery restrictions were eliminated, and all instances submitted to the mathematical heuristic. Table 3 shows the results compared with those with active constraints.

Table 3 - Results of Hybrid Mathematical Heuristics with Recovery Relaxation

		ID											
Group	**Variables**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
A	# Offsets	0	1	2	0	0	0	1	2	0	0		
	ΔCost (pp)	0	-4	-6	0	0	0	-4	-6	0	0		
	ΔTime (pp)	0	0	13	6	57	13	7	0	7	48		
B	# Offsets	3	9	4	3	6	3	9	4	3			
	ΔCost (pp)	-3	-4	-10	-4	-1	-3	-4	-10	-4			
	ΔTime (pp)	148	653	46	480	131	159	693	54	506			
C	# Offsets			0	0	2	0	2	0	5	20	4	25
	ΔCost (pp)			0	0	-5	0	-5	0	-6	-1	-10	-2
	ΔTime (pp)			6	-11	0	-20	0	-19	16	1640	712	511

The overall effect verified is the increase in execution time, as the optimizer starts to analyze more possibilities. Another effect was that many solutions have not reached a return to normality. This can be verified using the metric number of offsets occurred (# Offsets). Suppose that in the original schedule 3 aircrafts A320 should be at Congonhas airport at the end of recovery period. If there are 5 instead of 3, #Offsets is set to 2. When the final solution presents a shift, the cost of the solution is lower by up to 10%, what can represent significant savings – by not performing changes - for the airline whether it accepts this new situation in its schedule.

8. FINAL CONSIDERATIONS

The mathematical heuristic presented for the aircraft recovery problem proved to be effective, as it produced results close to the optimal for the instances considered (GAP_Atrib did not exceed 1% in most instances). It was also efficient, once it solved all instances in much less than the 30 minutes stipulated in the literature (Serrano *et al*., 2017) or even less than the 20 minutes considered by Petersen *et al*, 2012.

It was found that what makes an instance difficult is more the impact of disruptions than the size of the initial schedule. Also, it is possible to classify the disruptions regarding to the ability to clutter the schedule, making the recovery solution more difficult.

The reduction in airport capacity proved to be the most impactful disruption, followed by unscheduled maintenance and, finally, by flight delays and cancellations. This classification makes sense, as such disruptions consume more time of the aircraft, which is the scarcest resource of the schedule system. Delays and cancellations are generally absorbed by large turnaround times already present in the initial schedule of flights.

Changes in time discretization, or the use of heuristics rather than network flow model, are possibilities to explore in order to reduce execution times.

It is also worth taking into account more complex operational elements, such as transfers, use of airport slots and airline or passenger preferences.

The aircraft recovery problem solution is targeted at airlines. However, other operational optimization tools, such as A-CDM - Airport Collaborative Decision Making and TAM - Total Airport Management (Classen *et al*., 2017), could use ARP results as an input, or even be integrated to obtain broader solutions.

The results obtained allow to consider the proposed mathematical heuristic as a relevant contribution to the solution of real airline schedule problems.

It is worth exploring the use of this type of model for solving recovery problems of other transportation modes.

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